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Lorentz pole analysis of the reaction $\pi^+ n \to \omega p$

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Abstract. The high-energy behaviour of the reaction $\pi^+ n \to \omega p$ has been analysed in terms of the O(3,1) symmetry formalism. A good fit to the experimental data is obtained by choosing a ρ' trajectory in addition to the ρ trajectory, with the assumption that $\alpha_{\rho'} = \alpha_\rho - 1$. It is found that the ρ' trajectory belongs to the Lorentz quantum numbers $j_0 = 0$ and $j_0 = 1$.

1. Introduction

The reaction $\pi^+ n \to \omega p$ has been of some interest because of the fact that a simple ρ exchange fails to explain both the absence of a dip around $t = -0{\cdot}6(\text{GeV}/c)^2$ and also the non-zero value of the density matrix element ρ_{00} of ω. One of the ways of overcoming these difficulties is by introducing a secondary trajectory. Barwami (1966) has analysed this reaction using a B trajectory in addition to the ρ trajectory. We show in this paper that a good fit to the differential cross section data can also be obtained by choosing ρ' as the secondary trajectory. We have analysed this reaction in terms of the O(3,1) symmetry formalism of Delbourgo *et al.* (1967, to be referred to as DSS). The advantage of a ρ' over B is that the ρ' is included in this theory on the assumption that $\alpha_{\rho'} = \alpha_\rho - 1$. This preference of ρ' to B also implies the assumption that the B residue function is small. The validity of such an assumption was already shown for πN charge-exchange reactions (Antoniou *et al.* 1968), KN charge-exchange reactions (Samiullah and Srinivas Rao 1969 a) and for the charge-exchange reaction $\bar{p}p \to \bar{n}n$ (Samiullah and Srinivas Rao 1969 b). A ρ' exchange for this reaction is also suggested by Fox and Sertorio (1968). These authors consider the existence of the ρ' pole as a reasonably well-established feature. This reaction has also been analysed in terms of the absorption model (Jackson *et al.* 1965) and in terms of the Regge pole model with cuts generated by absorption (Henyey *et al.* 1968).

The number of amplitudes for this reaction is twelve, out of which only six are independent because of the invariance of the S matrix under parity transformation. When we further invoke the conservation of parity and G parity at each vertex (Cohen-Tannoudji *et al.* 1968) for the exchanged particle ρ, we find that only the amplitudes $\langle \frac{1}{2}0|T|\frac{1}{2}0\rangle$ and $\langle \frac{1}{2}1|T|\frac{1}{2}0\rangle$ contribute to the differential cross section. We now evaluate these amplitudes using the DSS formalism.

2. Evaluation of amplitudes

According to DSS, the conventional helicity amplitudes can be expanded in terms of a set of reduced amplitudes $T_{S'\lambda'S\lambda}(s, t)$ by the relation

$$\langle S_3\lambda_3 S_4\lambda_4|T|S_1\lambda_1 S_2\lambda_2\rangle = \sum_{S'\lambda'S\lambda} \langle S_4\lambda_4|S_2\lambda_2 S' - \lambda'\rangle T_{S'\lambda'S\lambda} \langle S_3\lambda_3 S\lambda|S_1\lambda_1\rangle \quad (1)$$

where $\langle S_i\lambda_i|S_j\lambda_j, S\lambda\rangle$ are the Clebsch–Gordan coefficients. These $T_{S'\lambda'S\lambda}(s, t)$ are further expanded in terms of a new set of reduced amplitudes $T_{J'\lambda S}$ by the relation

$$T_{S'\lambda'S\lambda}(s, t) = \sum_{J'} |t|^{\frac{1}{2}|\Delta|} \langle S'\lambda'||\Delta|\Delta, J'\lambda\rangle T^{S'}_{J'\lambda S} \quad (2)$$

where

$$-\Delta \equiv (\lambda_1 - \lambda_2) - (\lambda_3 - \lambda_4) = (\lambda_1 - \lambda_3) - (\lambda_2 - \lambda_4) = \lambda - \lambda'.$$

In the above expansion the factor $t^{\frac{1}{2}|\Delta|}$ is an approximation valid only for high energies and holds good for the energy of the reaction under discussion. The amplitude $T^{S'}{}_{\lambda S}$ when evaluated for an exchanged Lorentz pole yields the following result which is parity invariant (Antoniou *et al.* 1967):

$$T^{S'}_{J'\lambda S}(s,t) = \sum_{j_0=0}^{\min J'S} \tfrac{1}{2}(j_0^2 - \sigma^2)\beta^{S'}_{J'S}\left\{d^{j_0\sigma}_{S\lambda J'}(\mathscr{E}_t) + (-1)^{S+J'} d^{-j_0\sigma}_{S\lambda J'}(\mathscr{E}_t)\right\} \tag{3}$$

where $d^{j_0\sigma}_{S\lambda J'}$ are the matrix elements of the unitary representation O(3,1) and where

$$\cosh \mathscr{E}_t = \frac{s-u}{\{(2m_n^2 + 2m_p^2 - t)(2m_{\pi^+}^2 + 2m_\omega^2 - t)\}^{1/2}}.$$

The functions β are the residues of the amplitudes $T^{(j_0,\sigma,t)}_{J'\lambda S}$ at σ. From equations (1)–(3) we obtain the contributions of $j_0 = 0$ and $j_0 = 1$ for the helicity amplitudes. These are given in table 1.

Table 1. Contributions of $j_0 = 0$ and $j_0 = 1$ of the helicity amplitudes for the reaction $\pi^+ n \to \omega p$

Helicity amplitudes	$j_0 = 0$	$j_0 = 1$
$\langle\frac{1}{2}0\lvert T\rvert\frac{1}{2}0\rangle$	$-\sqrt{3}\beta_{11}{}^1 \dfrac{\alpha+1}{\alpha+2}\alpha(s')^\alpha$	$-2\sqrt{3}\dfrac{\alpha}{\alpha+1}\tilde{\beta}_{11}{}^1(s')^{\alpha-1}$
$\langle\frac{1}{2}1\lvert T\rvert\frac{1}{2}0\rangle$	$\lvert t\rvert^{1/2}(-6\beta_{11}{}^0 + \sqrt{6}\beta_{11}{}^1)\dfrac{\alpha+1}{\alpha+2}(s')^{\alpha-1}$	$\frac{1}{2}\lvert t\rvert^{1/2}\left\{\left(-3\tilde{\beta}_{11}{}^0 + \dfrac{\sqrt{3}}{2}\tilde{\beta}_{11}{}^1\right)\alpha + \dfrac{\sqrt{5}}{2}\beta_{11}{}^1\dfrac{\alpha(\alpha-1)}{\alpha+3}\right\}(s')^\alpha$

The O(3,1) expansion of $f_{++}(s,t)$ and $f_{+-}(s,t)$ are defined by the following formulae:

$$f_{++}(s,t) = \sum_{j_0} \frac{1}{2\pi \mathrm{i}}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \mathrm{d}\sigma (j_0^2 - \sigma^2) T^{j_0}_{++}(\sigma,t)\, \mathrm{d}^{j_0\sigma}_{S\lambda J'}(\mathscr{E}_t) \tag{4}$$

$$f_{+-}(s,t) = \sum_{j_0} \frac{2\sqrt{2}}{2\pi \mathrm{i}}|t|^{1/2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \mathrm{d}\sigma (j_0^2 - \sigma^2) T^{j_0}_{+-}(\sigma,t)\, \mathrm{d}^{j_0\sigma}_{S\lambda J'}(\mathscr{E}_t) \tag{5}$$

where $T^{j_0}_{++}(\sigma, t)$ and $T^{j_0}_{+-}(\sigma, t)$ are the partial wave amplitudes ($j = 0, 1$). Introducing the exchanged Lorentz pole ρ in the σ plane, we rewrite the expansions of $f_{++}(s, t)$ and $f_{+-}(s, t)$ in terms of the signature of the ρ pole following the method of Akyeampong *et al.* (1967):

$$\begin{aligned} f_{++}(s,t) = &-\sqrt{3}\beta_{11}{}^1 \frac{\alpha_\rho+1}{\alpha_\rho+2}\alpha_\rho \frac{1}{2}\frac{1}{\sin\pi\alpha_\rho}\{1 - \exp(-\mathrm{i}\pi\alpha_\rho)\}(s')^{\alpha_\rho} \\ &-2\sqrt{3}\frac{\alpha_\rho}{\alpha_\rho+1}\tilde{\beta}_{11}{}^1\frac{1}{2}\frac{1}{\sin\pi\alpha_\rho}\{1+\exp(-\mathrm{i}\pi\alpha_\rho)\}(s')^{\alpha_\rho-1} \end{aligned} \tag{6}$$

$$\begin{aligned} f_{+-}(s,t) = {}& |t|^{1/2}(-6\beta_{11}{}^0 + \sqrt{6}\beta_{11}{}^1)\frac{\alpha_\rho+1}{\alpha_\rho+2}\frac{1}{\sin\pi\alpha_\rho}\frac{1}{2}\{1+\exp(-\mathrm{i}\pi\alpha_\rho)\}(s')^{\alpha_\rho-1} \\ &+ \tfrac{1}{2}|t|^{1/2}\left[-3\tilde{\beta}_{11}{}^0\alpha_\rho + \frac{\sqrt{3}}{2}\tilde{\beta}_{11}{}^1\alpha_\rho + \frac{\sqrt{5}}{2}\beta_{12}{}^1\frac{\alpha_\rho(\alpha_\rho-1)}{\alpha_\rho+3}\right] \\ &\times \frac{1}{\sin\pi\alpha_\rho}\tfrac{1}{2}\{1-\exp(-\mathrm{i}\pi\alpha_\rho)\}(s')^{\alpha_\rho}. \end{aligned} \tag{7}$$

The differential cross section is then calculated from the formula

$$\frac{d\sigma}{dt} = \frac{1}{2\pi s^2}(|f_{++}|^2 + |f_{+-}|^2) \qquad (s \to \infty). \tag{8}$$

We have chosen the ρ trajectory (Hohler *et al.* 1966) to be $\alpha_\rho = 0{\cdot}57 + 0{\cdot}96t$.

3. Conclusions and discussion

Our results are in good agreement with the experimental data at 3·65 GeV/c (Benson 1967). Our analysis further confirms the mixing of j_0 values for ρ. With the assumption that $\alpha_{\rho'} = \alpha_\rho - 1$ we find that even ρ' belongs to both $j_0 = 0$ and $j_0 = 1$.

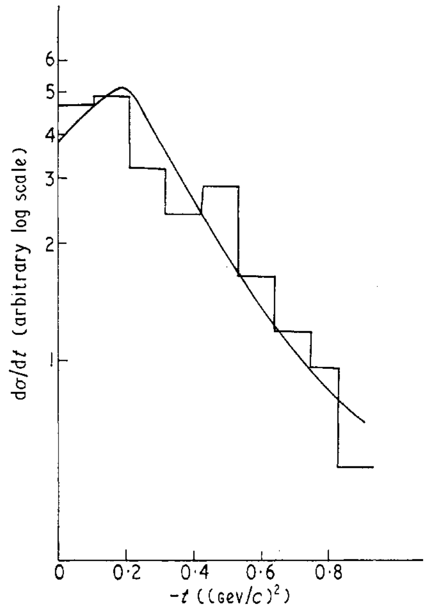

Figure 1. Differential cross section for the reaction $\pi^+n \to \omega p$ at 3·65 GeV/c. The full curve represents the result of O(3,1) symmetry formalism. Experimental data from Benson (1967).

This is a new result because it has so far been reported (Antoniou *et al.* 1968, Samiullah and Srinivas Rao 1969) that the ρ' had a unique j_0 value, i.e. $j_0 = 0$. It is found that the major contribution of ρ' is, however, from $j_0 = 0$ only. In fact, one can expect a mixing of j_0 values ($j_0 = 0$ and $j_0 = 1$) for ρ' as well if the quantum numbers of ρ' are assumed to be the same as for ρ. This is because the spinor representation obtained for the ρ particle (Komy *et al.* 1968) with a j_0 mixing of $j_0 = 0$ and $j_0 = 1$ is identical with that representation obtained for spin 1 particles by the Duffin (1938)–Kemmer (1939) β formalism. While fitting the experimental data we treated the residues β as parameters independent of t. The following are the values of the four independent β residues that were used in fitting the curve:

$$\beta_{11}{}^1 = \tilde{\beta}_{11}{}^1 = \frac{1}{\sqrt{3}}, \qquad \beta_{11}{}^0 = 15{\cdot}2, \qquad \tilde{\beta}_{11}{}^0 = 1, \qquad \beta_{12}{}^1 = \frac{5{\cdot}6}{\sqrt{5}}.$$

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